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A MEDIAL AXIS TRANSFORMATION FOR GRAYSCALE PICTURES

Shyuan Wang
Azriel Rosenfeld
Angela Y. Wu

Computer Vision Laboratory
Computer Science Center
University of Maryland
College Park, MD 20742

ABSTRACT

Blum's medial axis transformation (MAT) for binary pictures yields medial axis points that lie midway between opposite borders of a region, or along angle bisectors. This note discusses a generalization of the MAT in which a score is computed for each point P of a grayscale picture based on the gradient magnitudes at pairs of points that have P as their midpoint. These scores are high at points that lie midway between pairs of antiparallel edges, or along angle bisectors, so that they define a MAT-like "skeleton", which we may call the GRADMAT. However, this skeleton is rather sensitive to the presence of noise edges or to irregularities in the region edges.

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1. Introduction

In the early 1960's Blum [1] introduced the "medial axis transformation" (MAT) of a set S; this is basically the set of centers and radii of the maximal disks that are contained in S, or equivalently, the set of points of S whose distances to the complement $\overline{S}$ are local maxima, together with these distances. It is not hard to see that medial axis points tend to lie midway between opposite borders of S, or along the bisectors of angles formed by the borders. Thus these points constitute a kind of "skeleton" of S. For an introduction to the MAT, see [2], Section 9.2.3.

Blum's MAT is defined for a picture only after the picture has been segmented into S and $\overline{S}$. Several generalizations of the MAT to grayscale pictures have been suggested. We can define the gray-weighted length of a path as proportional to the sum of the gray levels at the points of the path; the gray-weighted distance between two points can then be defined as the lowest gray-weighted length of a path between them, and the gray-weighted MAT (GMAT) of a picture can be defined as the set of points whose gray-weighted distances to the set of 0's in the picture are local maxima, together with these distances [3]. Note that this definition still requires segmentation of the picture, since it treats 0's as "background" and regions of non-0 values as objects. Another generalization is based on finding maximal homogeneous disks in the given picture; the set of centers, radii, and

average gray levels of these disks defines a generalized MAT, called the SPAN ("Spatial Piecewise Approximation by Neighborhoods"), since this information can be used to generate approximations to the picture [4].

This note discusses a generalization of the MAT in which a score is computed for each point P of the picture based on the gradient magnitudes at pairs of points that have P as their midpoint. These scores are high at points that lie midway between pairs of antiparallel edges, or along angle bisectors, so that they define a MAT-like "skeleton", which we may call the GRADMAT. However, this skeleton is rather sensitive to the presence of noise edges or to irregularities in the region edges.

2. The GRADMAT

The basic idea of the GRADMAT is to compute a score for every P based on the gradient magnitudes at all pairs of points that have P as their midpoint. Evidently, this score will be very high at the center of a circle (or region that has a high degree of central symmetry); and it will also be high along the midline of a parallel-sided strip. There will be weaker responses at points that lie on local axes of symmetry, e.g., on angle bisectors, since such points are midway between at least one pair of edges.

These examples show that the GRADMAT is in many ways analogous to the MAT. However, it should be realized that the analogy is only partial. To see this, consider the GRADMAT of a horizontal rectangle, shown in Figure 1a (the algorithm used to compute this GRADMAT will be described later). The MAT of this rectangle consists of parts of the four angle bisectors and horizontal axis, as shown schematically in Figure 1b; the rest of the horizontal axis, and the vertical axes, do not belong to the MAT, since they are not centers of maximal neighborhoods. The GRADMAT, on the other hand, does give high values to the entire horizontal axis as well as the vertical axis (though these values become lower near the ends of the axes; see Figure 1a), since these axes are symmetrically located between the two pairs of sides of the rectangle. In this sense the GRADMAT more closely resembles

Blum's more recent idea of a multilevel MAT [5]. (In terms of Blum's "grassfire" metaphor, the higher levels of the MAT are generated by wavefronts that continue past one another after they collide, rather than "quenching" each other.)

To define the GRADMAT more precisely, we must specify just how the pairs of gradient magnitudes at positions symmetric with respect to P contribute to the score at P. The following points should be made in this connection:

a) Nonmaximum suppression in the gradient directions should be applied to the gradient magnitudes; in other words, for any point P, if one of P's neighbors in the direction of the gradient at P has a higher gradient magnitude than P does, P's gradient magnitude should be set to zero. If we do not thin the edge responses in the picture in this way, the GRADMAT values obtained will also be thick; in fact, if P is nearly halfway between two edges, nonmaximal responses from one or both edges will pair up to give P a relatively high score.

b) If two edges are an even distance apart (measured in pixel units), no point can be exactly midway between them. Thus if we thinned the edges so that they became one point thick, we would have to weaken the definition of "midway" to allow distances that differ by 1. Instead, we used a modified method of nonmaximum suppression that produces edges two points thick;

namely, the gradient magnitude at P is set to zero if a neighbor at distance 2 in the gradient direction has greater magnitude, but the magnitudes at the immediate neighbors are ignored. It can be verified that this yields edges two points thick except when there are exact ties in magnitude. Note that if two such edges are an odd distance apart, they give rise to a three-point thick GRADMAT, while if they are an even distance apart, the GRADMAT will be two points thick.

c) When P is midway between two edges, their contribution to P's score should be proportional to the minimum or product of their gradient magnitudes rather than to, e.g., their sum or average. In particular, P gets no score unless it is midway between two edge maxima, since the maxima are the only points at which nonzero gradient magnitudes remain. If we used the average, any strong edge would make a half-strength contribution to the score of every point even when there is no edge symmetrically located on the opposite side of the point, or when there is only a weak noise edge there.

d) An edge at point Q, say, should, contribute to the score at P only if the gradient direction at Q is roughly aligned with the direction from P to Q, so that the edge runs roughly perpendicular to the PQ direction. If we did not require this, we would obtain high GRADMAT values at points P that lie on straight edges, since such points have many pairs of edge points on both sides of them; but the edges at these points are collinear

with the direction from P, rather than perpendicular to it. One could reduce the GRADMAT values at points lying on edges by scaling the value at P in inverse proportion to the gradient magnitude at P; this will be illustrated in the next section. However, this would not prevent high GRADMAT values from arising at points that are collinear with a straight edge, whenever another edge lies somewhere on the other side of the point. Thus requiring the gradient direction at Q to be (say) within $\pm 45°$ of the direction PQ seems to be the best solution.

e) We should also require the pair of edges on opposite sides of P to face in opposite directions, i.e., P should be on the dark side or the light side of both of them. This is analogous to constructing the MAT of S and the MAT of $\overline{S}$, in the binary-valued case. In the examples in the next section, we have computed GRADMAT scores only for pairs of edges such that P is on the dark side of both; this is analogous to constructing the MAT of S (the "endoskeleton") but not that of $\overline{S}$ (the "exoskeleton" of S).

f) Even with these restrictions, it does not seem possible to prevent high GRADMAT values from being generated by edges that belong to two different boundaries. For example, if there are two dark objects, pairs of edges on the far sides of the objects will produce high GRADMAT values midway between these far sides, as illustrated in Figure 2a. Worse yet, if one dark

object surrounds another, i.e. if a dark object has a darker part, as in Figure 2b, high GRADMAT values will be produced by pairing off, e.g., the left side of the outer object with the right side of the inner one, and vice versa. We cannot eliminate these cases by allowing only the pair of edges closest to P in a given direction to contribute to P's value, since in a noisy picture this will often be a pair of noise edges. A somewhat more effective idea is to weight each edge's contribution according to its closeness to P; we could then allow only the edge in a given direction that makes the strongest contribution. If the objects all have the same contrast, this method rejects pairs of edges that do not belong to the same object boundary. However, if they have different contrasts, the results are harder to predict; close pairs of weak edges and more distant pairs of strong edges would compete, and the GRADMAT could change drastically if the relative strengths of these edges changed slightly, which is intuitively undesirable.

3. Experiments

In the following examples of GRADMATs, the Sobel edge operator was used to estimate the gradient magnitude and direction at each point, and nonmaxima of the gradient magnitude in the gradient direction were suppressed, as described in Section 2a-b. All gradient magnitudes were scaled relative to the strongest one in the picture; thus the scaled magnitudes are all in the range [0,1].

Contributions to the score of each point P were computed for all pairs of edge maxima on opposite sides of P at distance up to 25 pixels. In other words, if $P = (i,j)$, all pairs $(i+\alpha, j+\beta)$, $(i-\alpha, j-\beta)$ were examined for which $\sqrt{\alpha^2+\beta^2} \le 25$. The results were sorted by direction (i.e., by $\tan^{-1}\beta/\alpha$), to facilitate applying the direction criteria of Section 2d-e.

Figure 3 shows an airplane silhouette (a) and its gradient magnitudes (b). Figure 3c shows the resulting GRADMAT values, displayed as gray levels, scaled so the maximum score in the picture corresponds to black. In computing these values, the contribution of each pair of symmetric edge points is proportional to the product ($\le$ the sum) of the two gradient magnitudes, and an edge point at Q contributes to the score at P when P is on the dark side of Q and the gradient direction at Q is within $\pm 45°$ of the direction PQ. When this last condition is relaxed, high values are obtained nearly everywhere, as we see from Figure 3d. In these examples, the values are weighted in

inverse proportion to distance from P. Figure 3e shows results when no distance weighting is used; it is very similar to Figure 3c. Figure 4(a-e) shows analogous results for a much noisier picture, an infrared image of a tank. The last result (4e) is skeleton-like; in (4c) the skeleton is weakened by the inverse distance weighting.

4. Discussion and concluding remarks

Blum [5] has suggested that the MAT can be generalized to grayscale pictures by considering an expanding ring centered at every point P, and detecting when the ring hits edges. It seems likely that this would often yield confusing results; when the ring gets large, many different patterns of edges around the ring can give rise to the same total "score" at P, particularly if edges do not all have the same strength. In our approach, each pair of points symmetric with respect to P is considered individually; in effect, we are breaking up the ring into pairs of diametrically opposite points. As we have seen, even this more refined analysis of the edges surrounding P still yields noisy results.

The purpose of this paper was to define a plausible extension of the MAT to grayscale pictures. We have found, however, that the resulting GRADMAT is quite sensitive to noise; it gives skeleton-like results only in cases where the edges are very clean. In such cases, it would probably be safe to threshold the edges and use the ordinary MAT. The GRADMAT has the conceptual advantage of being defined, in principle, for unsegmented pictures; but it appears to be too sensitive to noise to be of practical use in most situations.

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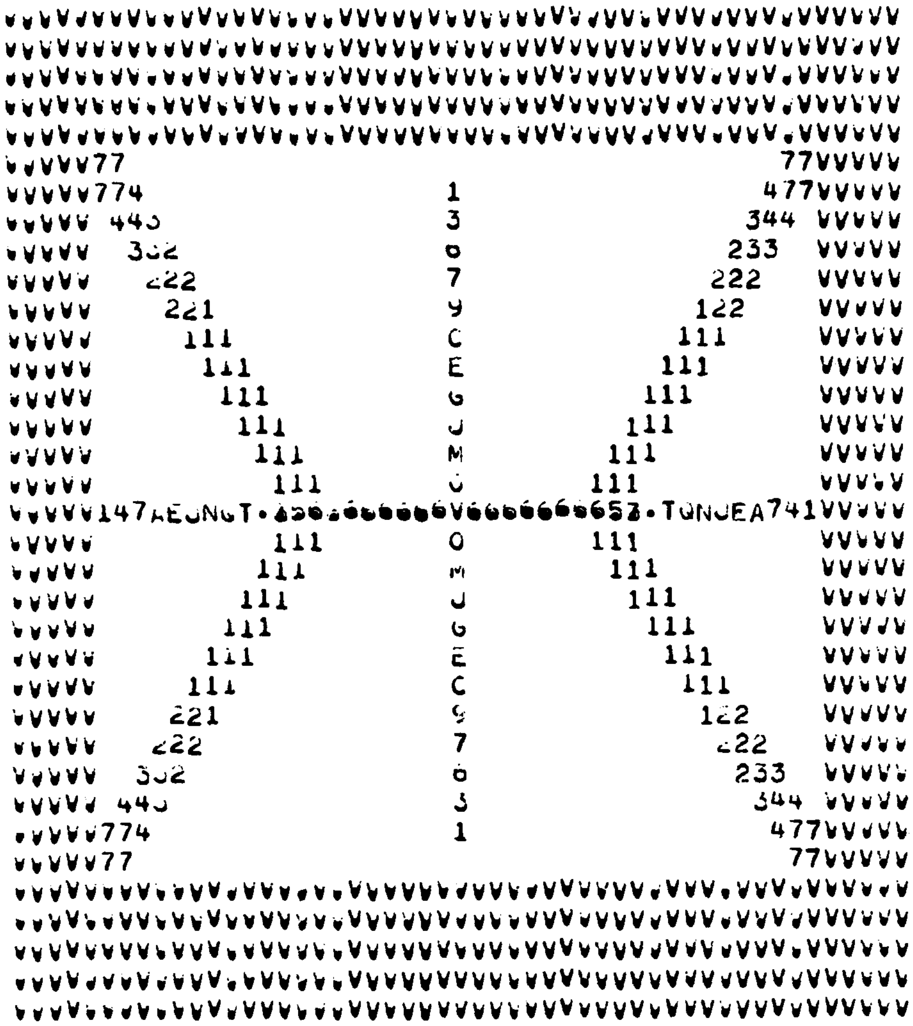
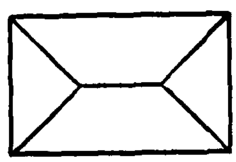


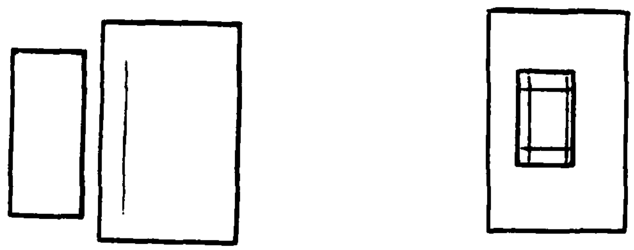
(a)

Key: blank,1,...,9,A,...,V = 0,1,...,31; same characters overstruck with periods = 32,...,63.



(b)

Figure 1. (a) GRADMAT of a rectangle; (b) Sketch of MAT of a rectangle.



(a) (b)

Figure 2. Sketches of GRADMAT points generated by (a) the far sides of two adjacent objects; (b) one object inside another. The GRADMATS of the objects themselves are not shown.

Fig. 3a-d

Fig. 3e

Fig. 4a-d

Fig. 4e

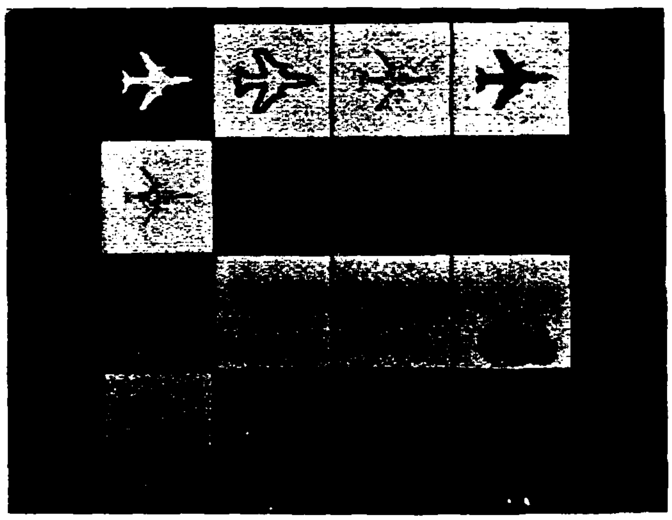


Figure 3. (a) Airplane silhouette. (b) Gradient magnitudes. (c) GRADMAT values. (d) Result of not requiring the gradient direction at a point to be within $\pm 45°$ of the direction to the point. (e) Result of $\overline{\text{not}}$ weighting the scores in inverse proportion to distance.

Figure 4. Analogous results for an infrared image of a tank.

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Blum's medial axis transformation (MAT) for binary pictures yields medial axi[s] points that lie midway between opposite borders of a region, or along angle bisectors. This note discusses a generalization of the MAT in which a score [is] computed for each point P of a grayscale picture based on the gradient magnitudes at pairs of points that have P as their midpoint. These scores are hig[h] at points that lie midway between pairs of antiparallel edges, or along angle bisectors, so that they define a MAT-like "skeleton", which we may call the GRADMAT. However, this skeleton is rather sensitive to the presence of nois[e]

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